Math help desk(4)

Department of Electrical Engineering

Contents:

- Laplace Transform
- Use of Laplace transforms to solve linear, constant coefficient differential equations.
- Inverse Laplace transforms.
- MATLAB applications

- The **Laplace transform** is a widely used integral transform in mathematics that transforms the mathematical representation of a function in time into a function of complex frequency

- In physics and engineering it is used for **analysis of linear time-invariant [LTI]systems** such as electrical circuits and mechanical systems.

- Given a simple mathematical or functional description of an input or output to a system, the Laplace transform provides an alternative functional description that often simplifies the process of analyzing the behavior of the system, or in synthesizing a new system. So, for example, Laplace transformation from the **time domain to the frequency domain transforms differential equations into algebraic equations** and convolution into multiplication.

Laplace Transform

Suppose that f(t) is a continuous function. The Laplace transform of f(t) is defined as

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

where $s = \sigma + j\omega$

Now, the integral in the definition of the transform is called an **improper integral** and it would probably be best to recall how these kinds of integrals work before we actually jump into computing some transforms

Remember that you need to convert improper integrals to limits.

Laplace transform of a function f(t) can be obtained with Matlab's function laplace.

Syntax: L =**laplace(f)**

$f(t)$	$F(s) = \int_0^\infty f(t)e^{-st}dt$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^3	$\frac{3!}{s^4}$
t^n	$\frac{n!}{s^{n+1}}$
$U_0(t)$	$\frac{1}{s}$
$U_c(t)$	$\frac{e^{-cs}}{s}$
e^{at}	$\frac{1}{s-a}$
$t e^{at}$	$\frac{1}{(s-a)^2}$
$t^2 e^{at}$	$\frac{2!}{(s-a)^3}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\delta(t)$	1
$\delta(t-a)$	$e^{-as}U_a(t)$
$\cos(bt)$	$\frac{s}{b^2+s^2}$
$\sin(bt)$	$\frac{b}{b^2+s^2}$
$t^2\cos(bt)$	$\frac{2s(s^2-3b^2)}{(b^2+s^2)^3}$
$t^2\sin(bt)$	$\frac{2b(3s^2-b^2)}{(b^2+s^2)^3}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$

Find the laplace transform:

$$f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$$

$$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$$

Inverse laplace transforms

$$f(t) = \mathcal{L}^{-1}\{F(s)\}$$

Find the inverse laplace transform:

$$F(s) = \frac{6s}{s^2+25} + \frac{3}{s^2+25}$$

$$F(s) = \frac{2-5s}{(s-6)(s^2+11)}$$

Solution of Differential Equations

But before proceeding into differential equations we will need one more formula. We will need to know how to take the Laplace transform of a derivative.

$$\mathfrak{L}\left\{f^{(n)}\right\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathfrak{L}\{y'\} = sY(s) - y(0)$$

$$\mathfrak{L}\{y''\} = s^2 Y(s) - sy(0) - y'(0)$$

Solve IVP's with Laplace Transforms

Examples:

Solve $y' - 3y = \exp(3x)$ $y(0)=0$

Solve $y'' + 5y' + 6y = 0$ $y(0)=2$, $y'(0)=3$

Solve $y'' - 10y' + 9y = 5t$ $y(0)=-1$, $y'(0)=2$

Solve $y'' + 3y' + 2y = \exp(-t)$ $y(0)=4$, $y'(0)=5$

Solve $2y'' + 3y' - 2y = t\exp(-2t)$ $y(0)=0$, $y'(0)=-2$

Thank you